#3 Hcri, Lisa, Faith

Chapter 3 Section 6

APPLICATIONS OF DERIVATIVES

A. Analysis of curves (3.6)

1. Determine the domain and range of the function.
2. Determine the intercepts, asymptotes and symmetry of the graph.
3. Find the x values for which the first derivative (f'(x)) and the second derivative (f"(x)) equal zero or are undefined. Use the x values to determine relative extrema and points of inflection.

Example:

Example:

$f(x) = \frac{x^2}{x^2-1}$

1) Domain = $\{x \mid x \neq 1\}$ Range = $(-\infty, 0] \cup (1, \infty)$

2) Intercepts – $x = (0,0)$

3) Symmetry – y-axis symmetry

4) Asymptotes – $(x=1)$ $(x=-1)$ $(y=1)$

5) $f'(x) = \frac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2} = \frac{-2x}{(x^2-1)^2}$ $\boxed{CP = (x=0)\ (x=\pm1)}$

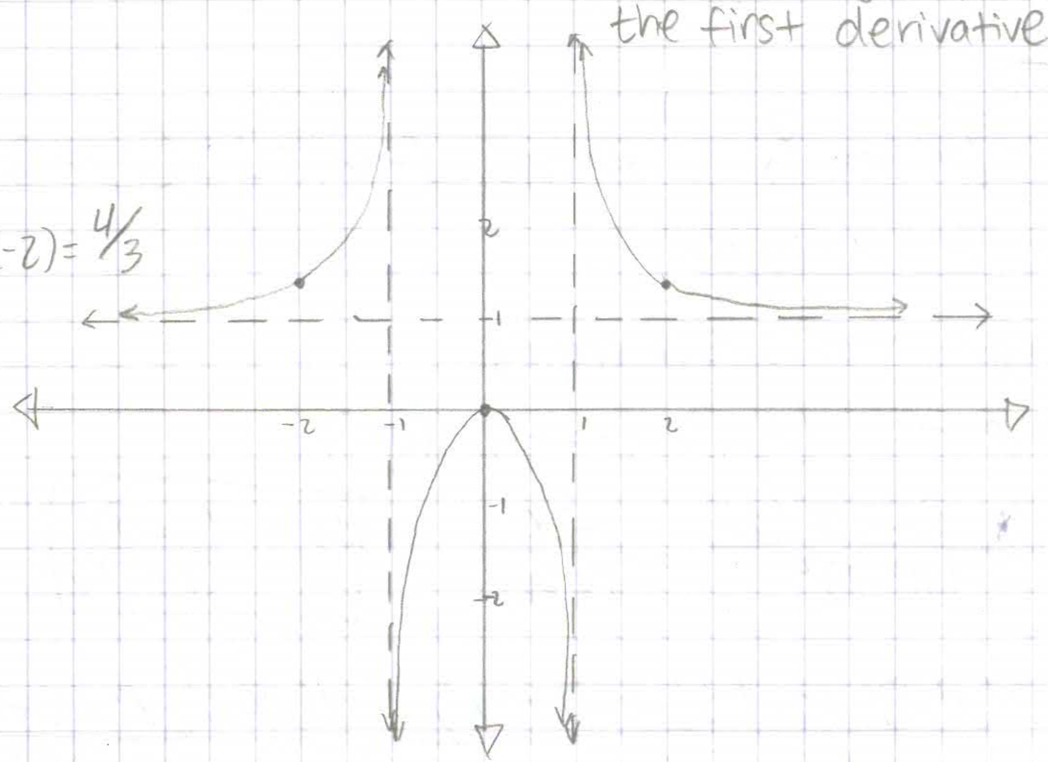
6) $f''(x) = \frac{-2(x^2-1)^2 + 2x(2(x^2-1)\cdot 2x)}{(x^2-1)^4} = \frac{2(\cancel{x^2-1})[-x^2+1+4x^2]}{(x^2-1)^{\cancel{4}\,3}}$

$= \frac{2(3x^2+1)}{(x^2-1)^3}$ $\boxed{PIP = (x=\pm1)}$

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
$f(x)$		und		0		und	
$f'(x)$	+	und	+	0	−	und	−
$f''(x)$	+	und	−	−	−	und	+
	I CU	V.A.	I CD	x-int	D CD	V.A.	D CU

rel max
because it goes from + to − on the first derivative.

$f(-2) = 4/3$

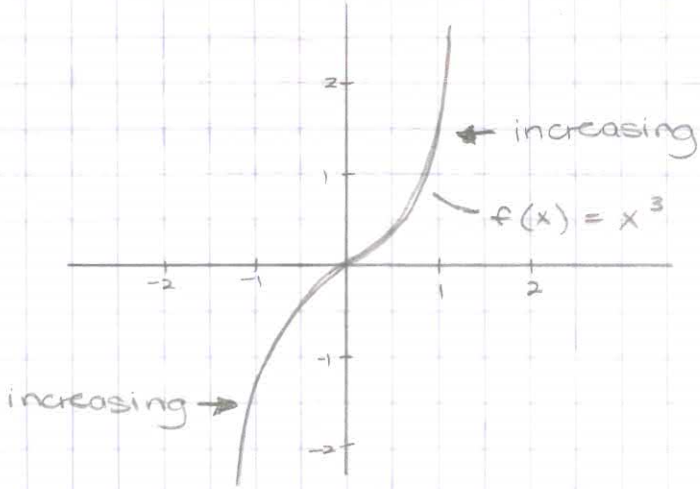


Chapter 3 Section 3

B. Monotonicity (3.3)

A function is strictly monotonic on an interval if it is either increasing (stays positive) on the entire interval or decreasing (stays negative) on the entire interval.

Example: $f(x) = x^3$ is strictly monotonic on the entire real line because it is increasing on the entire real line.



Chapter 3 Section 4

C. Concavity (3.4)

Let f be differentiable on an open interval (a,b).

Concave Up (Open up)

- If f''(x) (the second derivative) > 0 on (a,b), then f is concave up on (a,b).

Concave Down (Open down)

- If f''(x) (the second derivative) < 0 on (a,b), then f is concave down on (a,b).

It is also known as the rate of change of the slope (is the slope increasing or decreasing?) and can be found by using the second derivative.

Example: Determine the open intervals on which the graph of $f(x) = \frac{x^2+1}{x^2-4}$ is concave upward or downward

Solution: $f(x) = \frac{x^2+1}{x^2-4}$

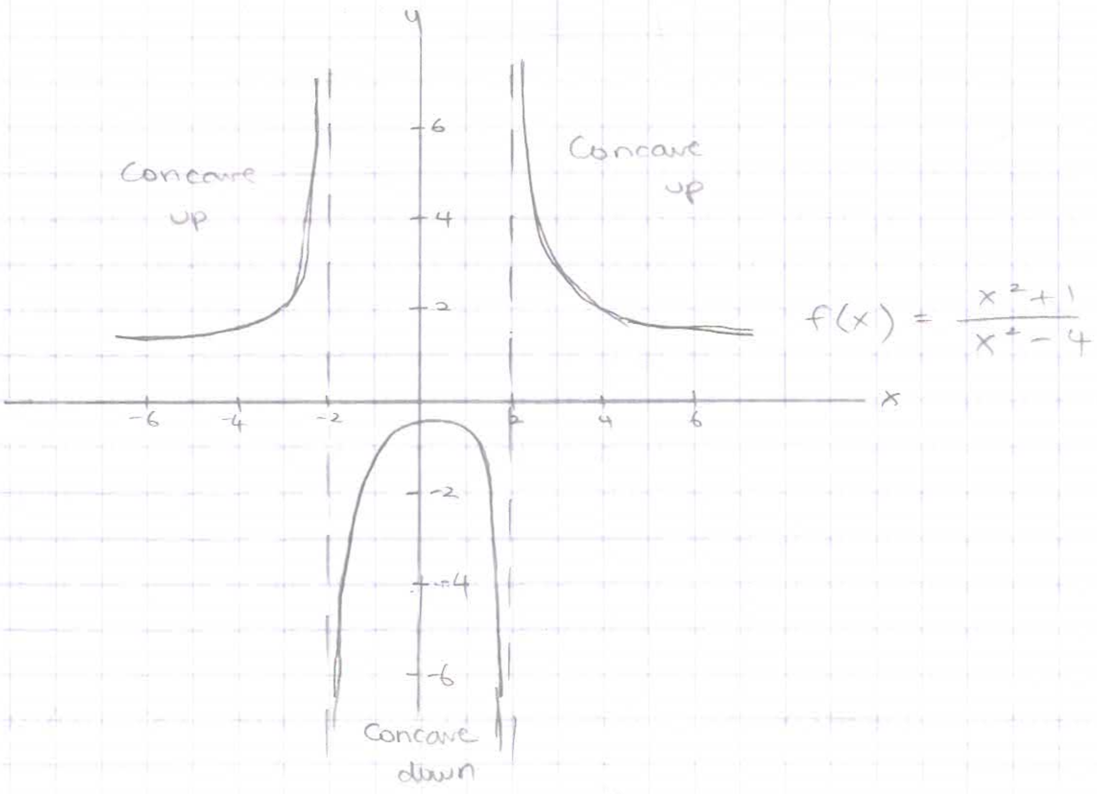
$$f'(x) = \frac{(x^2-4)(2x) - (x^2+1)(2x)}{(x^2-4)^2}$$

$$= \frac{-10x}{(x^2-4)^2}$$

$$f''(x) = \frac{(x^2-4)^2(-10) - (-10x)(2)(x^2-4)(2x)}{(x^2-4)^4}$$

$$= \frac{10(3x^2+4)}{(x^2-4)^3}$$

Interval	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
Test Value	$x = -3$	$x = 0$	$x = 3$
Sign of $f''(x)$	$f''(-3) > 0$	$f''(0) < 0$	$f''(3) > 0$
Conclusion	Concave Upward	Concave Down	Concave upward



chapter 3
section 7

D. Optimization (3.7) – the process of finding a minimum or maximum value as a solution to a problem.

1. Optimization Process:

   a. Find an equation for the value you are trying to optimize.

   b. Find the critical points.

   c. Use the critical points and the end points to determine the maximum value.

example

A rectangular page is to contain 24 in of print. The margins at the top and bottom of the page are to be 1½ inches. And the margins on the left and right are to be 1 in. what should the dimensions of the page be so that the least amount of paper is used?

$A = (x+3)(y+2)$ (area to be minimized)

area inside the margin is $24 = xy$

$\hookrightarrow y = 24/x$

$A = (x+3)\left(\frac{24}{x}+2\right) = 30 + 2x + \frac{72}{x}$ $\quad x > 0$

$\frac{dA}{dx} = 2 - \frac{72}{x^2} = 0 \quad x^2 = 36 \quad x = 6, \cancel{-6} \rightarrow \boxed{x=6}$

$24 = 6y \quad \boxed{y=4}$

Dimentions of page

$\hookrightarrow \boxed{9\text{in} \times 6\text{in}}$

$x + 3 = 9$ inches
by
$y + 2 = 6$ inches

chapter3 section1

2. Extreme Value Theorem: If f is continuous on intervals [a,b], then f has both a minimum and a maximum on the interval.

3. Relative (Local) Extrema – only occurs at critical numbers (when the first derivative = 0 or is undefined) and is either the maximum or the minimum on some specific open interval.

4. Absolute (Global) Extrema – can occur at both critical numbers and potential inflection points (when the second derivative = 0 or is undefined) and is either the maximum or the minimum on the entire graph.

Example Find the abs. min and abs. max of $f(x) = x^2+2x-4$ in the interval $[-2,1]$

$f(x) = x^2+2x-4$

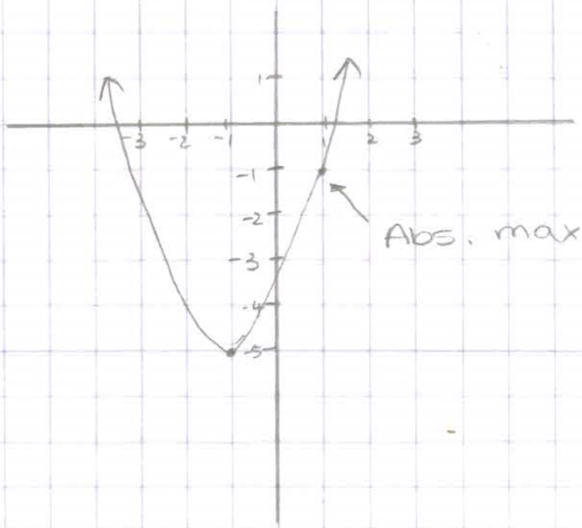
$f'(x) = 2x+2 = 0$

$x = -1$

$f(-1) = -5$ Abs. min

$f(-2) = -4$

$f(1) = -1$ Abs. max



Abs. max but NOT relative max

Chapter 2
section 6

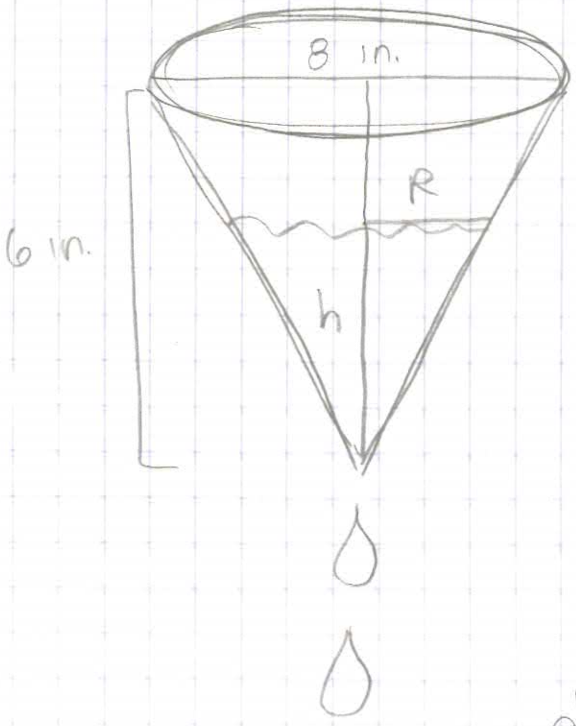
E. Related Rate Problems (2.6)

1. List all the variables and rates with values for each. (Note: Rates are listed as derivatives, often with respect to time.)
2. Find an equation(s) that relate the different quantities.
3. Differentiate the equation implicitly.
4. Solve for the unknowns and answer the question in complete sentences.

Example:

example:

A conical paper cup 8 in. across the top and 6 in. deep is full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 in. deep?



$$\frac{dV}{dt} = \frac{2\ in^3}{min}$$

$$h = 3\ in \qquad \frac{dh}{dt} = ?$$

$$V = \tfrac{1}{3}\pi R^2 h \qquad \frac{R}{h} = \frac{4}{6} \quad R = \frac{2}{3}h$$

$$V = \frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h$$

$$V = \frac{4}{27}\pi h^3$$

$$\frac{dV}{dt} = \frac{4}{9}\pi h^2\left(\frac{dh}{dt}\right)$$

$$2 = \frac{4}{9}\pi(3)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{2}{4\pi} = \frac{1\ in.}{2\pi\ min}$$

chapter 2
section 5

F. Implicit Differentiation (2.5) is the procedure used when the equation expressed is not in terms of y but rather is a function in terms of x.

1. Differentiate both sides of the equation with respect to x.
2. Collect all terms involving dy/dx on one side of the equation and move all other terms to the other side of the equation.
3. Factor dy/dx out of one side of the equation.
4. Solve for dy/dx.

Example

Find dy/dx given that $y^3+y^2-5y-x^2=-4$

1) $\frac{d}{dx}\left[y^3+y^2-5y-x^2\right]=\frac{d}{dx}\left[-4\right]$

2) $\frac{d}{dx}(y^3)+\frac{d}{dx}(y^2)-\frac{d}{dx}(5y)-\frac{d}{dx}(x^2)=\frac{d}{dx}(-4)$

3) $3y^2\frac{dy}{dx}+2y\frac{dy}{dx}-5\frac{dy}{dx}-2x=0$

4) $\frac{dy}{dx}(3y^2+2y-5)=2x$

5) $\boxed{\frac{dy}{dx}=\frac{2x}{(3y^2+2y-5)}}$

Chapter 2
Section 2

G. Rate of Change (2.2) is the slope or change in position over change in time, depending on what kind of application problem. Rate of Change is represented by the derivative of a function of one variable with respect to another. Therefore, the position function is derived to get the velocity function, which can then be derived to get the acceleration function.

Position ($s(t) = 1/2gt^2 + v_0(t) + s_0$) → Velocity ($v(t) = gt + v_0$)→ Acceleration ($a(t) = g$) (derivative going from left to right)

Example: A feather is dropped in a vacuum from a height of 300 feet. Find the average velocity.

a. Over the first two seconds.
b. During the 2nd second.
c. Between 1 to 1.1 second
d. At t = 1 second

a) $s(t) = -16t^2 + 300$

$$\frac{s(2) - s(0)}{2 - 0} = \frac{236 - 300}{2 - 1} = -32 \text{ ft / sec}$$

b) $$\frac{s(2) - s(1)}{2 - 1} = \frac{236 - 284}{2 - 1} = -48 \text{ ft / sec}$$

c) $$\frac{s(1.1) - s(1)}{1.1 - 1} = \frac{280.64 - 284}{.1} = -33.6 \text{ ft / sec}$$

d) $v(t) = -32t$

$v(1) = -32$ ft / sec

Example: A rocket is propelled straight up so that in t seconds, it reaches a height of

$f(t) = 2t^2 + t$

a. Find the instantaneous velocity at any time t.

b. Find the average velocity during the first 78 feet of flight.

c. Find the instantaneous velocity after rising 210 feet.

a) $f(t) = 2t^2 + t$

$\boxed{v(t) = 4t + 1}$

b) $\frac{\Delta s}{\Delta t} = \frac{78}{6} = 13$ ft / sec

c) $210 = 2t^2 + t$

$t = 10$

$v(10) = 4(10) + 1 = 41$ ft / sec

H. Geometric Interpretation and Slope Fields (6.1):

Chapter 6 section 1

A. Slope field is a direction field for a differential equation.

- General solution: a family of solutions to a diff. eq. (contains + C)
- Particular solution: single solution to a diff. eq. based on initial conditions

Example:

Draw a slope field for the following differential equations.

1) $\frac{dy}{dx} = x + 1$